# The Open Systems View

(based on joint work with Stephan Hartmann)

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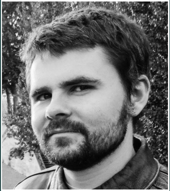

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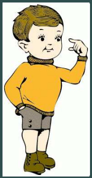

M. E. C.

M. E. C. and Hartmann, S. (eds.),
*Open Systems: Physics, Metaphysics, and Methodology*,
Oxford University Press (in preparation)

With chapters by:

- Stephan Hartmann & MEC; Emily Adlam; Luis C. Barbado & Časlav Brukner; Eddy K. Chen; Gemma De las Cuevas; George Ellis; Doreen Fraser & Adam Koberinski; Sean Gryb & David Sloan; Bill Harper; James Ladyman & Karim Thébault; Olimpia Lombardi; Wayne Myrvold; Jørn Kløvfjell Mjelva, Josh Quirke & Alistair Wilson; Lev Vaidman, and David Wallace, and others.

Outline:

1. Standard quantum theory (**ST**) and the closed systems view
2. Reasons to worry about the closed systems view
3. The general quantum theory of open systems (**GT**) and the open systems view
4. The principle of complete positivity

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## The closed systems view

(1) Metaphysical motivation: Isolated systems are the proper subject matter of science.

(2) Associated with the methodology that models all phenomena fundamentally in terms of closed systems.

A "view" is similar to a "stance" (van Fraassen) but unlike a stance includes (1) in addition to (2).

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Strictly speaking no system (except, perhaps, the whole universe) can really be isolated. How do we model open systems in **ST**?

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Closed systems view of an open system:

- $\mathcal{S}$'s interaction with its environment is described in terms of its being coupled with a separate system $\mathcal{E}$ such that $\mathcal{S} + \mathcal{E}$ form an isolated system.

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- Yields (effective) non-unitary dynamics for $\mathcal{S}$:

$$\dot{\rho}_{\mathcal{S}} = (\mathcal{L}_{u} + \mathcal{L}_{n\text{-}u})\, \rho_{\mathcal{S}},$$

$$\text{with:} \quad \mathcal{L}_{n-u}\, \rho = \frac{1}{2} \sum_i \left( [L_i\, \rho, L_i^\dagger] + [L_i, \rho\, L_i^\dagger] \right),$$

where the $L_i$ are (bounded) operators.

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- Black hole physics gives us (prima facie) reasons to motivate the idea that the dynamics of certain systems are fundamentally open (Hawking, 1976).
- Global unitary evolution is hard to square with important recent approaches to quantum gravity (Oriti, 2021, sec. 3.1).

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- Although this typically involves the methodological assumptions associated with the closed systems view, it isn't the dynamics of $\mathcal{S} + \mathcal{E}$, but the dynamics of $\mathcal{S}$, that we take ourselves to have successfully described when we do this.
- Thus there is a clear empirical motivation to extrapolate from the dynamics of open systems rather than from the dynamics of closed systems.

Cf. Newton's 4th rule of reasoning: "In experimental philosophy, propositions gathered from phenomena by induction should be considered either exactly or very nearly true notwithstanding any contrary hypotheses, until yet other phenomena make such propositions either more exact or liable to exceptions." (Quoted in Harper 2011, ch. 7)

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Both (neo-)Everettian and (neo-)Bohrian approaches are ontologically committed to open systems, or so we argue (MEC & Hartmann, 2024b)

- See extra slides at the end of this talk.

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## The open systems view

(1) Metaphysical motivation: The proper subject matter of science is systems that are, in general, open.

(2) Associated with the methodology according to which one need not model phenomena in terms of closed systems. The influence of the environment on a system is represented fundamentally in terms the dynamical equations that we take to govern its evolution (but not necessarily the evolution of $\mathcal{S} + \mathcal{E}$).

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  - The environment is not represented as a separate system; its influence is represented in the dynamical equations that govern the evolution of $\mathcal{S}$.
- Physical state of $\mathcal{S}$ represented by a density operator, $\rho$
- Time-evolution of $\rho$ is governed by a dynamical map, $\Lambda_t$:

$$\Lambda_t \rho_0 = \rho_t$$

  - $\Lambda_t$ acts on the state space of $\mathcal{S}$ (not on $\mathcal{S} + \mathcal{E}$).

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Consequence:

$$\dot{\rho}_{\mathcal{S}} = \left(\mathcal{L}_{\mathrm{u}} + \mathcal{L}_{\mathrm{n\text{-}u}}\right) \rho_{\mathcal{S}}$$

- Non-unitary dynamics in general; unitary dynamics as a special case.

There is more to **GT** than the Lindblad equation. We can consider relaxing some of its assumptions. For instance,

- Markov (Barandes, 2023)
- Semigroup, continuity (Wolf & Cirac, 2008).
- Complete positivity: The focus of the rest of this talk.

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Complete positivity:

Our claim:

- Complete positivity is an expression of the closed systems view. “Not completely positive” (NCP) maps do not make physical sense on the closed systems view.
- But on the open systems view, one should deny complete positivity the status of a fundamental physical principle.

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- State change of $\mathcal{S}$ in the presence of $\mathcal{E}$: $\rho_{\mathcal{S}} \mapsto \mathrm{tr}_{\mathcal{E}}(U\rho_{\mathcal{S}} \otimes \rho_{\mathcal{E}}U^\dagger) = \rho'_{\mathcal{S}} = \Lambda\rho_{\mathcal{S}}$

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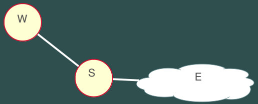
Complete positivity:




Consider the system $\mathcal{S} + \mathcal{W}_n$ evolving in the presence of $\mathcal{E}$, such that:

- $\mathcal{W}_n$: A system of dimensionality $n$ not currently interacting with $\mathcal{S}$, but which may have interacted with it in the past.
- $\mathcal{W}_n$ evolves trivially.

$\rho_\mathcal{S} \mapsto \mathrm{tr}_\mathcal{E}(U\rho_\mathcal{S} \otimes \rho_\mathcal{E} U^\dagger) = \Lambda\rho_\mathcal{S}$

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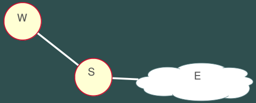


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Problem: Requiring that $\Lambda$ be positive on $\mathcal{S}$ does not guarantee that $\Lambda \otimes I_n$ is positive on $\mathcal{S} + \mathcal{W}_n$.

- I.e., $\Lambda \otimes I_n$ will map some of the states in $\mathcal{H}_{\mathcal{S}+\mathcal{W}_n}$ to unphysical states (i.e., that yield negative probabilities for the results of measurements on $\mathcal{S} + \mathcal{W}_n$).

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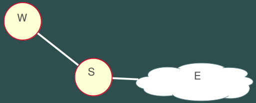


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Solution: Require that $\Lambda \otimes I_n$ be positive for all $\mathcal{W}_n$.

"One may reasonably doubt this argument. It is very powerful magic: $\mathcal{W}$ sits apart from $\mathcal{S} + \mathcal{E}$ and does absolutely nothing; by doing so, it forces the motion of $\mathcal{S}$ to be completely positive with dramatic physical consequences ..." (Pechukas, 1994).

More concrete reasons to be skeptical (Shaji & Sudarshan, 2005):

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- Important because it can be shown (Jordan et al., 2004, pp. 13–14) that the contracted dynamics of a system $\mathcal{S}$ in the presence of $\mathcal{E}$ is describable by a completely positive map only if $\mathcal{S}$ is initially not entangled with $\mathcal{E}$.

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- If we enforce complete positivity then it would seem to follow that no valid physical description of the dynamics of $\mathcal{S}$ can be given when it is initially entangled with $\mathcal{E}$.
  - See extra slides at the end of this talk.

## Not completely positive (NCP) maps:

It is precisely for setups like these that an NCP map will make sense.

- When $\mathcal{S}$ and $\mathcal{E}$ are entangled, then it follows that it is impossible for $\mathcal{S}$ to be (for instance) in a pure state, or in general in any state that is not a valid partial trace over the combined state of $\mathcal{S} + \mathcal{E}$.

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- Such states that are outside of the 'compatibility domain' of an NCP map will be ill-described by it.
- But as long as such a map is completely positive in relation to all of the actually possible states of $\mathcal{S}$ in a given setup, it seems that there is no reason not to use it to describe the dynamics of $\mathcal{S}$ (MEC & Myrvold, 2013, sec. 5).

A better argument for imposing complete positivity as a fundamental physical principle:

> *A system-theoretic description of an open system has to be considered as phenomenological; the requirement that it should be derivable from the fundamental automorphic dynamics of a closed system implies that the dynamical map of an open system has to be completely positive.*
> (Raggio & Primas, 1982, p. 435, our emphasis).

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- Yields (effective) non-unitary dynamics for $\mathcal{S}$:

$$\dot{\rho}_{\mathcal{S}} = (\mathcal{L}_{u} + \mathcal{L}_{n\text{-}u})\, \rho_{\mathcal{S}},$$

with: $$\mathcal{L}_{n-u}\, \rho = \frac{1}{2} \sum_i \left( [L_i\, \rho, L_i^\dagger] + [L_i, \rho\, L_i^\dagger] \right),$$

  where the $L_i$ are (bounded) operators.
- Guaranteed by Stinespring's dilation theorem (Stinespring, 1955), which assumes complete positivity.

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So **GT**, unlike **ST**, allows for fundamental non-unitary evolution, and is, in this sense, a more general dynamical framework than **ST** (despite not adding anything to the Hilbert space formalism).

> ~~A system-theoretic description of an open system has to be considered as phenomenological; the requirement that it should be derivable from the fundamental automorphic dynamics of a closed system implies that the dynamical map of an open system has to be completely positive.~~
> (Raggio & Primas, 1982, p. 435).

Conclusion: We should reject complete positivity as a fundamental physical principle.

For more on the open systems view, see MEC & Hartmann (2023, 2024a) and our forthcoming book:

M. E. C. and Hartmann, S. (eds.),
*Open Systems: Physics, Metaphysics, and Methodology*,
Oxford University Press (in preparation)

With chapters by:

- Stephan Hartmann & MEC; Emily Adlam; Luis C. Barbado & Časlav Brukner; Eddy K. Chen; Gemma De las Cuevas; George Ellis; Doreen Fraser & Adam Koberinski; Sean Gryb & David Sloan; Bill Harper; James Ladyman & Karim Thébault; Olimpia Lombardi; Wayne Myrvold; Jørn Kløvfjell Mjelva, Josh Quirke & Alistair Wilson; Lev Vaidman, and David Wallace, and others.

# Extra slides

# Approaches to ST that take it to be "complete"

(see MEC & Hartmann 2024b)

$$
\begin{aligned}
|\psi\rangle_{\mathcal{S}} &= \alpha|b_1^+\rangle + \beta|b_1^-\rangle \\
&= \alpha'|b_2^+\rangle + \beta'|b_2^-\rangle.
\end{aligned}
$$

What does this mean on a (neo-)Bohrian interpretation?

- Coupling the degrees of freedom of $\mathcal{S}$ to those of a further system $\mathcal{M}$ will yield a collection of unitarily-related conditional probability distributions over the possible outcomes of an assessment of $\mathcal{M}$ as described with respect to a particular basis $b_m$.

"In the treatment of atomic problems, actual calculations are most conveniently carried out with the help of a Schrödinger state function, from which the statistical laws governing observations obtainable under specified conditions can be deduced by definite mathematical operations. It must be recognized, however, that we are here dealing with a purely symbolic procedure, the unambiguous physical interpretation of which in the last resort requires a reference to a complete experimental arrangement." (Bohr, 1958, pp. 392–393, our emphasis).

(neo-)Bohrian interpretation:

- Notice that $\mathcal{S}$ is conceived of here as an open system (even when its state is described by a state vector), but since open systems dynamics are not fundamental in **ST**, we require a larger Hilbert space (including the degrees of freedom of both $\mathcal{S}$ and $\mathcal{M}$) to represent it as such.

(neo-)Bohrian interpretation:

- Notice that $\mathcal{S}$ is conceived of here as an open system (even when its state is described by a state vector), but since open systems dynamics are not fundamental in **ST**, we require a larger Hilbert space (including the degrees of freedom of both $\mathcal{S}$ and $\mathcal{M}$) to represent it as such.
- **ST** is about open systems, on a (neo-)Bohrian interpretation, despite being formulated from the closed systems view (which it inherits from classical mechanics).

The (neo-)Everett interpretation:

- **ST** is as much about subsystems described by density operators as it is about composite systems described by state vectors (see, e.g., Wallace and Timpson's (2010): "Spacetime state realism.")

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- More principled argument: Corresponding to any given probability distribution over pure states of $\mathcal{B}$ (i.e., to any given density operator for $\mathcal{B}$), one can always find a pure state of some larger system $\mathcal{A} + \mathcal{B}$ from which that probability distribution can be derived.
- All such purifications are "essentially the same" (D'Ariano et al., 2017, p. 171).

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- All such purifications are "essentially the same" (D'Ariano et al., 2017, p. 171).
- $\rho_{\mathcal{S}}$, expressed as a decoherent mixture of various states corresponding to the elements of an eigenbasis of $\rho_{\mathcal{M}}$ is just as objective a description of everything there is, relative to the degrees of freedom included in our representation of $\mathcal{S}$ and to the given eigenbasis, as the universal state vector $|\Psi\rangle_{\mathcal{S}+\mathcal{M}}$.

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  - Ultimately the (neo-)Everett interpretation is committed to quantum theory, not necessarily the closed systems view of quantum theory.

What about many worlds?

$$\rho = p\,|\psi_1\rangle\langle\psi_1| \; + \; (1-p)\,|\psi_2\rangle\langle\psi_2|.$$

- The fact that the different terms, $|\psi_1\rangle\langle\psi_1|$ and $|\psi_2\rangle\langle\psi_2|$ are by definition decoherent makes it unproblematic, irrespective of whether $\rho$ evolves unitarily, to identify them with independently evolving worlds.
- This is even clearer than the FAPP story one needs to give in the pure state, unitary, case.

# Complete Positivity and Entanglement

(see MEC & Myrvold 2013)

$$\rho_S \mapsto \Lambda\rho_S =$$

$\rho_S \mapsto \Lambda\rho_S = \qquad \Phi\rho_S$

Assignment map: $\Phi\rho_S = \rho_{SE}$

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Unitary on $SE$

$$\rho_S \mapsto \Lambda\rho_S = \mathrm{tr}_E(U\ \Phi\rho_S\ U^\dagger)$$

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Trace over $E$

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Unitary on $SE$: *CP*
Trace over $E$: *CP*

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*Suppose* $\Phi$ *is:*

'Linear': $\Phi\left[\lambda\rho_1 + (1-\lambda)\rho_2\right] = \lambda\Phi\rho_1 + (1-\lambda)\Phi\rho_2$,
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*Then* $\Phi$ *is CP*

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Suppose $\rho_{SE}$ entangled:

- $\Phi$ (hence $\Lambda$) may be NCP
- Compatibility domain: $\rho_S \in \mathcal{H}_S$ s.t. $\Phi\rho_S \geq 0$ (hence: $\Lambda\rho_S \geq 0$)

[Submitted on 7 Nov 2018 (v1), last revised 25 Jun 2021 (this version, v2)]

# A post-quantum theory of classical gravity?

Jonathan Oppenheim

We present a consistent theory of classical gravity coupled to quantum field theory. The dynamics is linear in the density matrix, completely positive and trace-preserving, and reduces to Einstein's theory of general relativity in the classical limit. As a result, the dynamics doesn't suffer from the pathologies of the semi-classical theory based on expectation values. The assumption that general relativity is classical necessarily modifies the dynamical laws of quantum mechanics -- the theory must be fundamentally stochastic in both the metric degrees of freedom and in the quantum matter fields. This allows it to evade several no-go theorems purporting to forbid classical-quantum interactions. The measurement postulate of quantum mechanics is not needed since the interaction of the quantum degrees of freedom with classical space-time necessarily causes decoherence. We first derive the most general form of classical-quantum dynamics and consider realisations which have as its limit deterministic classical Hamiltonian evolution. The formalism is then applied to quantum field theory interacting with the classical space-time metric. One can view the theory as fundamental or as an effective theory useful for computing the back-reaction of quantum fields on geometry.

Comments: "It's very difficult to find a black cat in a dark room, especially if there is no cat."
Subjects: **High Energy Physics - Theory (hep-th)**; Quantum Physics (quant-ph)
Cite as: arXiv:1811.03116 **[hep-th]**
(or arXiv:1811.03116v2 **[hep-th]** for this version)
https://doi.org/10.48550/arXiv.1811.03116

An early attempt to couple classical gravity to quantum mechanics was made in [51] using the Aleksandrov-Gerasimenko bracket [52, 53]

$$\frac{\partial \sigma}{\partial t} \stackrel{?}{=} -i[\mathbf{H}, \sigma] + \frac{1}{2}\Big(\{\mathbf{H}, \sigma\} - \{\sigma, \mathbf{H}\}\Big) \tag{4}$$

However, the dynamics it generates leads to negative probabilities [51, 54]. Unless we are prepared to modify the Born rule, or equivalently, our interpretation of the density matrix, any dynamics must be trace-preserving (TP) to conserve probability, and be *completely positive (CP)*, for probabilities to remain positive. So, although the dynamics of Equation (4) makes a frequent appearance in attempts to couple quantum and classical degrees of freedom [55–57] (c.f. [58]), and while it may give insight in some regimes, it cannot serve as a fundamental theory. Other attempts, such as using the Schroëdinger-Newton equation [59, 60] (c.f. [61, 62]), suffer from the problem that the equations are non-linear in the density matrix, and thus will lead to superluminal signalling [63] and a breakdown of the statistical mechanical interpretation of the density matrix[7].

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