Analogies in formation, action, and motion

The fascinating case of spontaneous symmetry breaking

Rocco Gaudenzi

.: University of Verona :.

UNIVERSITÀ
di VERONA

MAX-PLANCK-GESELLSCHAFT

Introduction & overview I

- **Subject.** Heuristic process leading to concept of spontaneous symmetry breaking (SSB)
  - Long and progressive process spanning over three decades (ca. 1930-60)
  - Accretion, around a **core analogy**, of motivations, methods and concepts from three fields
    - Nuclear physics, particle physics, solid state physics
  - SSB occurs *analogously* in solid bodies as well as the universe at large
    - Loss of one of the symmetries featured by system's dynamical law and normally inherited by its states
    - Ground state (GS) transitions to a lower symm. accompanied by emergence of specific excitations
    - Occurs when changing boundary conditions without the intervention of symmetry-breaking external fields
    - *Ex.* Phase transitions (ferromagnets, gas to liquid/solid transitions, superconductors etc.)
- **Analogical making.** Looking at and reasoning on sth. as if sth. else
  - Surrogative and symmetric/invertible operation (analogon and analogue may switch)

Introduction & overview II

- **Core analogy.** Universe $\leftrightarrow$ solid body

  Universe; GS (*vacuum*) + exc. states $\leftrightarrow$ Solid; GS + exc. states

  1. Vacuum as a non-interacting solid (Fermi gas, fully symmetric and continuous exc. spectrum)
     - 1929. Dirac sea; theory of relativistic electron
  2. Non-/interacting solid (nuclear matter, plasmas, superconductors) as "apparent" vacuum
     - 1950. Nambu's apparent vacuum; effect of nuclear matter on pion self-energy
     - 1954. Non-perturbative quantum electrodynamics in solid media
     - 1959. Reformulation of superconductivity
  3. Vacuum as a superconductor (non-symmetric GS and gapped excitation spectrum)
     - 1960. Nambu and Jona's superconductor model of elementary particles

  - Each of the three uses of analogy enabled a transfer of concepts (C) and methods (M)
    - 1. C (vacuum as a dielectric solid, polarisation, ionisation etc.)
    - 2. C & M (self-energy, vacuum polarisation, vertex part, renormalisation, non-pert. approx. etc.)
    - 3. C & M (non-pert. renormalisation, SSB $\rightarrow$ non-trivial QFT vacuum, phase transitions etc.)

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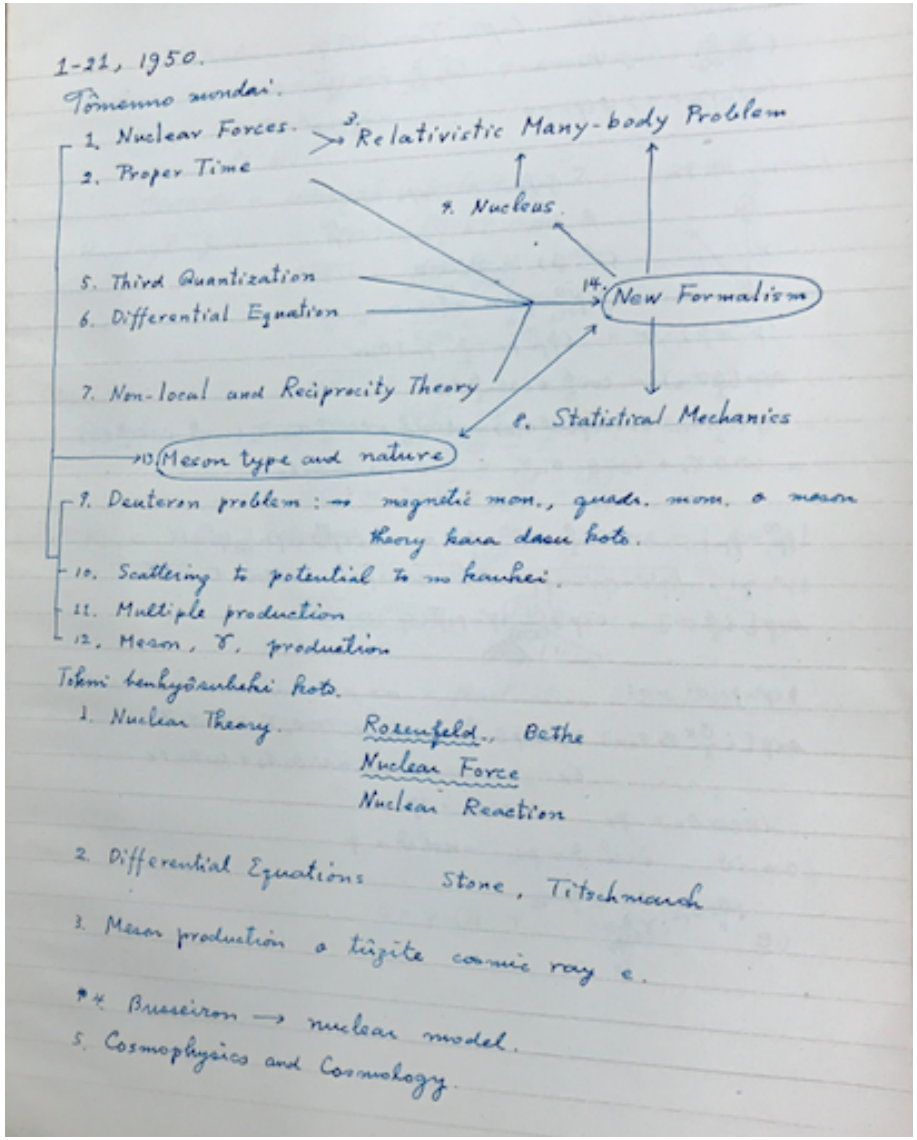
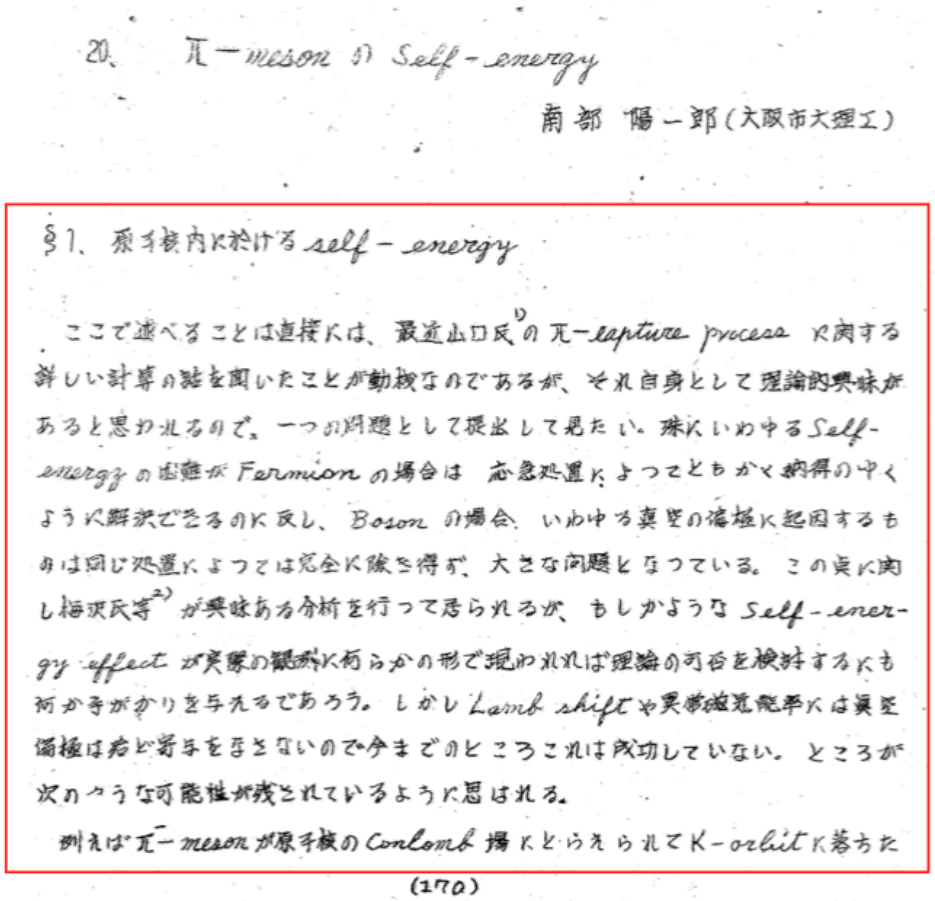
Original overarching goal. Note from January 21, 1950



The first stone. Brief communication of April 1, 1950



20. π⁻-meson の Self-energy

南部 陽一郎（大阪市大理工）

§1. 原子核内における self-energy

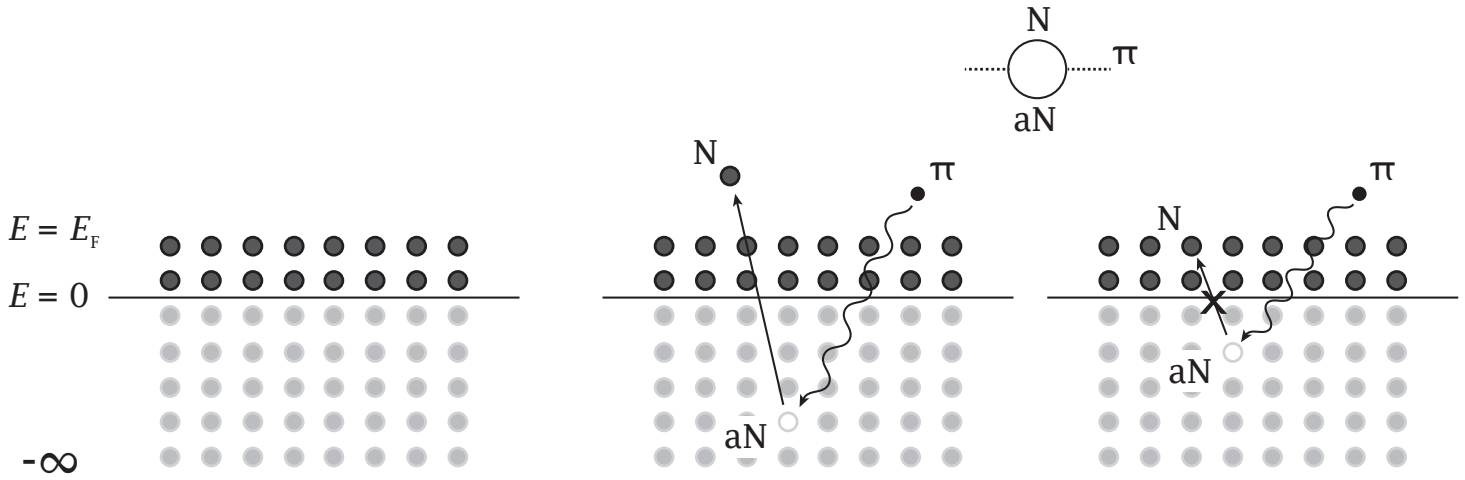
ここで述べることは直接には、最近山口氏[1]の π⁻-capture process に関する詳しい計算の話を聞いたことが動機なのであるが、それ自身として理論的興味があると思われるので、一つの問題として提出して見たい。殊にいわゆる Self-energy の困難が Fermion の場合は 応急処置によってともかく納得の中くように解決できるのに反し、Boson の場合、いわゆる真空の偏極に起因するものは同じ処置によっては完全に除き得ず、大きな問題となっている。この点に関し梅沢氏等[2]が興味ある分析を行って居られるが、もしかような Self-energy effect が実際の現象に何らかの形で現われれば理論の可否を検討するにも何か手がかりを与えるであろう。しかし Lamb shift や異常磁気能率には真空偏極は殆ど寄与をなさないので今までのところこれは成功していない。ところが次のような可能性が残されているように思はれる。

例えば π⁻-meson が原子核の Coulomb 場にとらえられて K-orbit に落ちた

(170)

- "The direct motivation of what I will discuss is Mr. Yamaguchi's detailed calculation but because it is theoretically interesting for its own sake, I will propose it as a problem"
  - Effect of the surrounding nuclear media on nuclear reactions and nuclei properties
  - Difference in probability of pion capture in simple and complex nuclei

STEP 1. Nambu repurposes the Dirac Sea



- 1929. Material interpretation of vacuum as a Fermi gas (Dirac sea)
  - Forbids, by Pauli principle, electrons from falling into negative energy states
  - Explains electron-positron pair generation from vacuum as photon inducing polarisation
  - Dirac sea (or vacuum as dielectric medium) acts as phenomenological scaffold for QED

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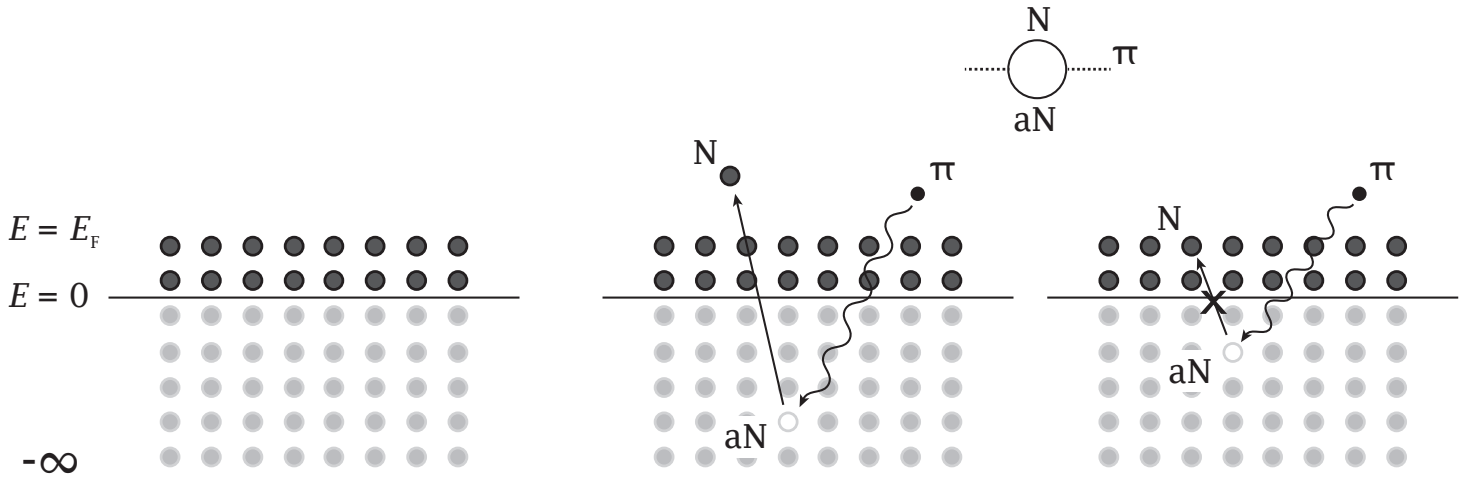

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- 1950. Interpretation of nuclear matter as extra piece of Dirac sea (apparent vacuum)
  - Forbids generation of low-energy virtual nucleon-antinucleon pairs $\rightarrow$ modifies pion self-energy
  - Modifies dielectric/"di-mesic" properties of vacuum $\leftrightarrow$ properties of particles immersed in it
  - Acts as scaffold for QED and mesodynamics in solid media
    - Fermi sea and Dirac sea both obey Pauli principle
    - Real particles act on particles in the vacuum $\rightarrow$ treatable with analogous tools of QFT

STEP 2. Generalization to interacting media (nuclei and plasmas)

I was suggested by your articles ... to examine the [collective] oscillations of the plasma as a preliminary to many important problems, such as the properties of the nucleus, superconductivity, the origin of solar noise... The essential idea of my treatment is the same as yours, but I make use of the recently developed ideas of quantum electrodynamics: interaction representation and renormalization.

A more elegant way to look at the idea here ... is to look at the state in which all levels up to the Fermi energy are occupied as an "apparent vacuum". (Nambu to Pines, Jan. 1952)

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PHYSICAL REVIEW VOLUME 94, NUMBER 3 MAY 1, 1954

The Collective Description of Many-Particle Systems (A Generalized Theory of Hartree Fields)

TOICHIRO KINOSHITA AND YOICHIRO NAMBU
Institute for Advanced Study, Princeton, New Jersey

(Received October 1, 1953; revised manuscript received December 29, 1953)

- 1952-54. Non-perturbative math. framework (GHF) for particles in interacting media
  - QFT version of self-consistent field method from many-body physics
  - Calculation of additional finite *non-perturbative* contribution to the self-energy given by *interacting* apparent vacuum and renormalisation of particles' properties
  - Application to many-body systems: electrons in ionic lattice (plasmas) and mesons in nuclei

STEP 3. Generalisation to media w/ broken symmetry phase

- 1954-56. Nambu works on weak and strong interactions
- 1956. Theoretical explanantion of superconductivity by Bardeen and colleagues
  1. Dynamical law: phonon-mediated electron-electron attractive interaction
     - Formation of Cooper pairs and lowering of GS energy in superconducting phase
     - Emergence of gap in energy spectrum between GS and single-particle excitations
     - Emergence of collective excitations (plasmons)
  2. Gauge-symmetric dynamical law $\rightarrow$ GS violating gauge symmetry/charge conservation

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  2. Gauge-symmetric dynamical law $\rightarrow$ GS violating gauge symmetry/charge conservation
- 1956-60. Nambu extends his non-perturbative framework to apply it to superconductors
  - Gap treated as a self-consistent self-energy of the electron. Logical implications:
    - When renormalised, electron acquires new symmetry-breaking property entirely dynamically

      Impossible to obtain with perturbative method as that preserves symmetry
    - Emergence of collective excitations restoring charge conservation

      Expression for the current of renormalised electrons has an additional term
  - Superconductivity as the first instance of spontaneous symmetry breaking
    - Gauge invariance, broken symmetric GS (gap), emergence of collective excitations logically implied
    - Symmetric dynamical law can have non symmetric solutions

STEP 4. Superconductor model of elementary particles

$$(1) \quad J^{\mathrm{sc}} = J_{\mathrm{Q}} - 2\phi\, e\partial_\mu f$$

$$(2) \quad J^{\mathrm{wi}} = J_{\mathrm{N}} - 2\, m\, \partial_\mu \pi$$

- 1959. *Formal* analogy: charge current in SC and nucleon chiral currents in weak interactions:
  - Ad hoc additional term in (2) to ensure conservation of chirality for massive particles

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A 'SUPERCONDUCTOR' MODEL OF ELEMENTARY PARTICLES AND ITS CONSEQUENCES by Y. Nambu (University of Chicago)†

Superconductivity	Elementary particles
free electrons	bare fermion (zero or small mass)
phonon interaction	some unknown interaction
energy gap	observed mass (nucleon)
collective excitation	meson bound nucleon pair
charge	chirality
gauge invariance	γ_5−invariance (rigorous or approximate)

- 1960 Reinterpretation of the ad hoc relation of elementary particles in light of SC
  - Transfer of relation: chiral inv., broken symmetric GS (mass), emergence of collective exc.s
  - Non-symmetric vacuum as non-perturbative solution of chiral symmetric dynamical law
  - Massless particle aquires mass purely dynamically like superconducting gap
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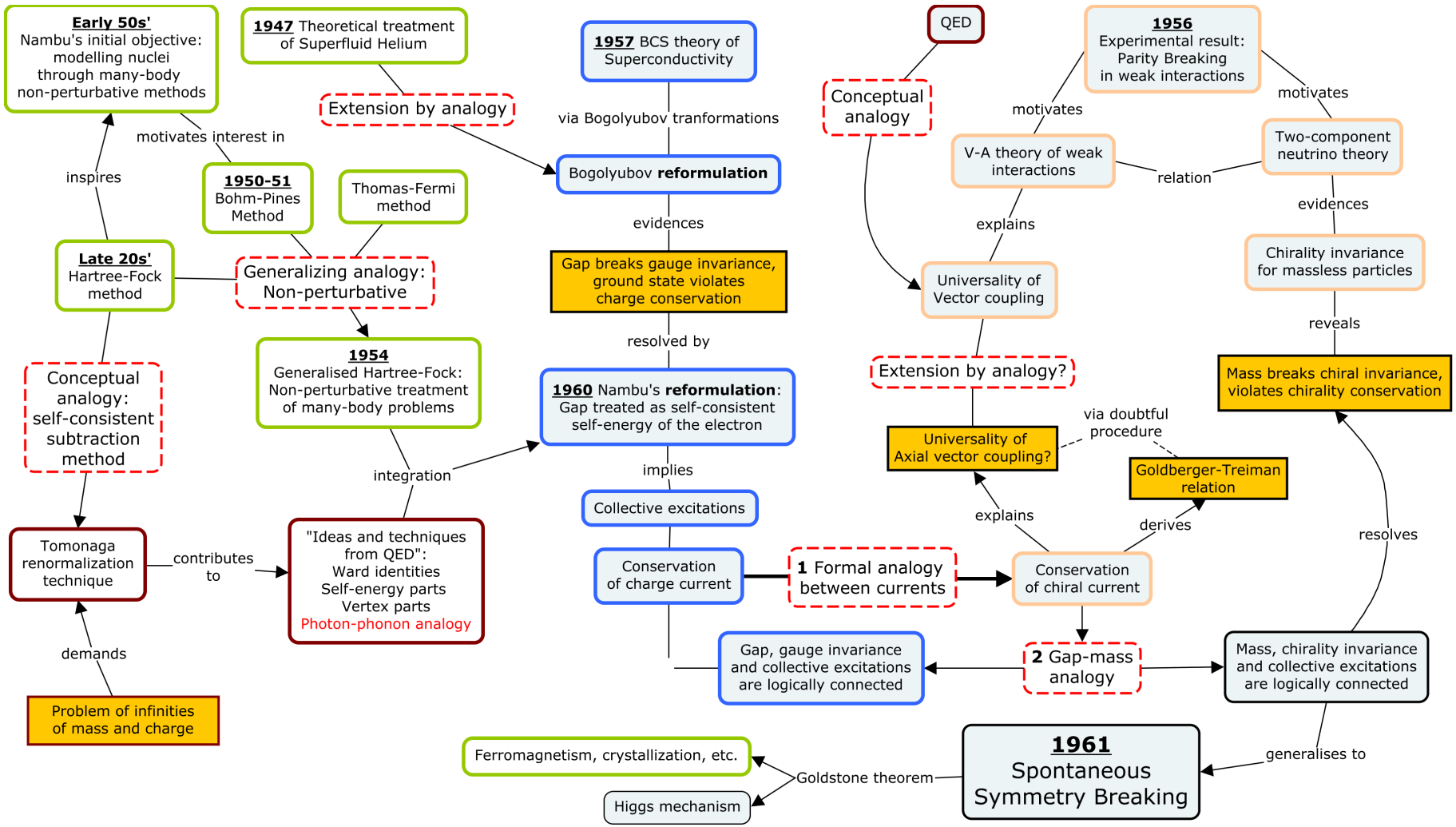
  - Energy spectrum: massive fermions and mesons
- Vacuum is again regarded as a solid, not as a Fermi gas, but as a superconductor

The universe is a superconductor?

Nambu at International Conference of high-energy physics (Kiev, 1959):

> *I would like to draw attention to an analogy between the problem of chirality invariance in field theory and that of gauge invariance in superconductivity [. . .]. In this analogy the mass of a particle corresponds to the energy gap in a superconductor. The recently proposed description of superconductivity . . . is not gauge invariant due to such a gap. But several people have managed to interpret the current theory of superconductivity in such a way that it is invariant under gauge transformations. The invariance under chiral transformations could also be treated in a similar way. [. . .]*
> *It therefore seems that by* **studying superconductivity there is the possibility of learning something about elementary particles**.

The conceptual network leading to the conceptualisation



Discussion I

- Action and motion of core/background analogy
  (interactions and properties of universe $\leftrightarrow$ solid body)
  - Analogy whose terms are implicit or unspecified (metaphor; framework)
    - Opened a new dimension of thought more than establishing specific correspondences
    - Platform for development of smaller and triggering analogies
  - Characterised by heterogenesis of intents
    - Addressing saturation of nuclear force by considering effect of surrounding medium on pions
    - Turned out to do more and different than foreseen
    - Space of motives, problems and possible uses reshapes itself and opens up on the way
      (Creation of novel affordances, evolutionary biology)
  - Enabled transfer conceptual and method. resources back/forth; solid-state and particle physics
    - Made visible logical relations not visible within one domain (like math. transform)
    - Three inversions of source and target domain
    - Acted alternately as a scaffold to develop one and the other field

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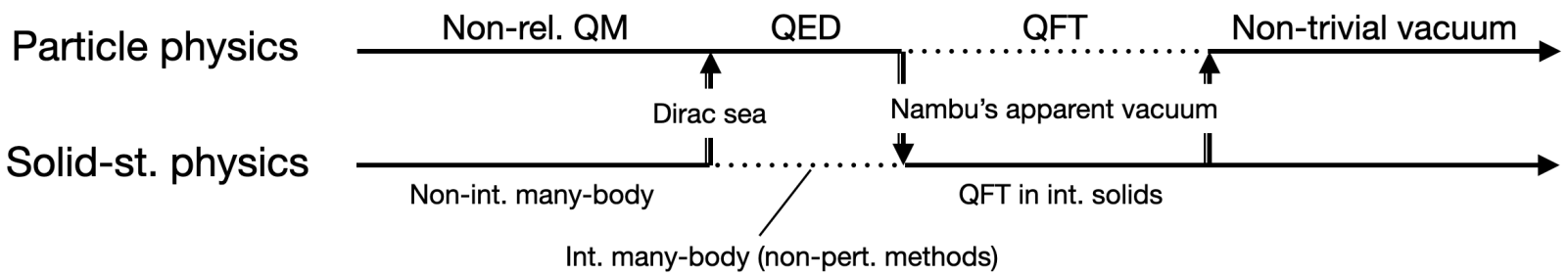

Discussion II

- Disciplinary cross-fertilisation
  - Elementary particle theory (QFT) lent part of the formalism to discover SSB in superconductors
  - Solid-state physics:
    1. Lent motivation and conceptual and methodological base for non-perturbative framework
    2. Illuminated SSB in the more abstract and less intuitive realm of particle physics
  - Enabled to solve analogous open problems in both and arrive at a concept applicable to both
- Superconductor as a scale model of the universe
  - Systems with physical manipulability used to infer on one with no manipulability
    - Universe and superconductor both underwent a phase transition lowering symmetry
    - GSs of both described only as non-perturbative non-symmetric solution of symmetric dynamical equation

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- Analogy enabling factors
  - Contextual
    - Familiarity with problems/methods from particle physics, physics of solids and nuclei
    - Influence of Yukawa's substantialistic approach and Tomonaga's use of analogy
  - Cultural and personal idiosincrases
    - Tendency to materialise abstract concepts and think of them in concrete terms

      "... He did not seem to make logical connections but associations: in the middle of one of his classes a string became a vortex and then a flux tube" M. Mukerjee (Nambu's former student)

Thank you for your attention!

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