Analogies in Quantum Field Theory

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Analogies in Physics an Beyond

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Motivation

Analogies show up everywhere in science, which raises many philosophicial questions, e.g. concerning their role and value.

> "Analogies in physics and beyond will examine the circulation of models within and across the sciences. Leading contemporary epistemologists will analyze the process of exchanging scientific knowledge from one field to another, detailing the challenges that often stand in the way of such transfers, as well as the opportunities that emerge from current research. The overarching aim is to dig deeper into the creative side of scientific thinking while tackling the philosophical questions that successful exchanges may raise."

I will consider a case study – spontaneous symmetry breaking in quantum field theory – to address some of the philosophical questions and to (hopefully) learn some more general lessons about analogies.

Outline

I. Analogies in Science

Analogies and Analogical Reasoning

- We distinguish between analogies and analogical reasoning (Bartha).
- Analogies identify a similarity between a source (S) and target (T).
- Analogical reasoning is about providing reasons for a property (widely construed) of T based on the properties of S.

Example 1

(P1) There is life on Earth. (P2) The Earth and Mars are similar.
(C) There is life on Mars.

Example 2

(P1) Water waves propagate through a medium. (P2) Sound waves propagate through a medium.
(C) Light waves propagate through a medium.

Analogies and Analogical Reasoning

- These arguments sometimes work and sometimes do not work.
- Some of them can be reconstructed as (enumerative) inductive arguments or other kinds of arguments.

Example

(P1) If conditions X hold, then there is life. (P2) The conditions X hold on Mars.
(C) There is life on Mars.

- So one can get around the explicit mention of the analogy, but one needs something like similarity to motivate P1.
- Other arguments may not be reconstructed without explicit reference to an analogy in the premises.

The Role of Analogies and Analogical Reasoning

- There are different kinds of analogies.
- A distinction: formal (or structural) and material analogies.
- In the sciences, analogies are used for different purposes.
- Most importantly, perhaps, they have an important heuristic role.
- The epistemic role of analogies is more controversial.
    1. Which role do analogies play in scientific explanations?
    2. Do analogies play a role in the business of confirming a theory?

The Role of Analogies and Analogical Reasoning

Questions

How can one explore the epistemic role of analogies and analogical arguments? What justifies an analogical argument?

- One needs a philosophical theory of analogy and analogical reasoning.
- There are several proposals in the literature. Some of these theories are formal, others are material.
- From the start, one can be skeptical that these theories do much.
- One immediately faces the notorious problem of induction (even simple enumerative induction cannot be justified) and Goodman's new riddle of induction (which raises doubts about the value of similarity judgments)?

Formal Theories of Analogies

- Following Aristotle and J.S. Mill, Mary Hesse and Paul Bartha proposed formal theories of analogies.
- Aristotle and Mill: Consider what S and T have in common ("positive analogy") and what not ("negative analogy"). What can we infer about what is not established yet ("neutral analogy")?
- Hesse worked these ideas out and also stressed, besides the horizontal relations, vertical relations.

	Earth (S)		**Mars** (T)
		Known similarities:	
↑	orbits the sun		orbits the sun
vertical	has a moon	← **horizontal** →	has moons
	revolves on axis		revolves on axis
↓	subject to gravity		subject to gravity
		Inferred similarity:	
	supports life	⇒	may support life

Formal Theories of Analogies

- Hesse also distinguished between formal (or structural) analogies and material (or physical) analogies.
- Her focus is on horizontal relations and material analogies, but she also stressed the importance of causal vertical relations.
- The articulation model of Bartha builds on and extends these ideas. In particular, Bartha stresses vertical relations (in S) which can then be used to generalize from S to T.
- Bartha also developed (formal) criteria to assess the quality of an analogical inference and he tried to justify them.
- Defenders of the material theory of analogy raise doubts about this project.

The Material Theory

- According to Norton's material theory of analogy, there is no formal theory and all the work is done by local facts about the source and the target – facts of analogy.
- Norton criticises the specific proposals of Hesse and Bartha and reconstructs his own examples.
- However, Norton's material theory has many problems itself (see Genta 2020):
  1. It does not apply to all cases (esp. in physics).
  2. The notion of similarity needs to be developed.
  3. It is not normative.
  4. It faces counter examples (and so it is also not descriptive).

Other Functions of Analogies

- Science is arguably not only about coming up with confirmed predictive theories that motivate interventions.
- There are various other functions and it has been argued that analogies (and metaphors and models of various kinds) play an important role here.
  1. Explanation
  2. Understanding
  3. Interpretation (see the work of Daniela Bailer-Jones)
- There is also a literature in cognitive science (computational models etc.) that explores these functions of analogies.

Physics

- There are plenty of examples for the use of analogies in physics.
- Think about the mechanical models of Maxwell and Kelvin. Physicists like to transfer concepts, ideas and models from one area to another.
- These models have to be adapted to the new environment, which is sometimes not easy or uncontroversial, but it often works like this. To conquer the unknown (or not yet understood) one borrows whatever promises to work from another (supposedly analogous) field. This can be a model, a concept, or a problem solving strategy.
- In this way, one arguably strikes a balance between a conservative and a progressive research strategy (or so Bartha argues).
- The analogy does not necessarily have to come from ordinary life (recall Lakoff and Johnson´s Metaphors we live by), but can also come from other theories which one has under control.
- The analogy can also be more liberal.

II. Quantum Field Theory and the Standard Model

The Standard Model

- Quantum field theory grew out of quantum mechanics.
- The starting point was that one wanted to quantize not only the particles (such as electrons) but also the fields (that mediate the interaction between the particles).
- This project turned out to be harder than one thought: infinitely many modes, infinities,. . . how to proceed?
- An early analogy was that one knew how to quantize the harmonic oscillator. This helps on the way to early fomulations of QED.
- QED was then completed in the late 1940ies, addressing the problem of the infinities ("renormalization"). It is a gauge theory (with the massless photon as the gauge boson) and it allows for a perturbative treatment (the coupling constant $\alpha \approx 1/137$ is small).
- QED then served as the model for the development of two further theories that then form the standard model of particle physics.

The Standard Model

- The electroweak theory unifies electromagnetic and weak interactions.
- Interestingly, one could only come up with a renormalizable theory of weak interactions if one takes electromagnetism on board.
- Quantum chromodynamics (QCD) is the fundamental theory of strong interactions.
- Here we will focus on QCD.

Quantum chromodynamics (QCD)

- This theory is much more complicated than the others.
- The strong interactions are short range.
- There is a host of strongly interacting particles ("hadron zoo"), some have very low but non-vanishing masses (like the pion), other particles have larger masses, such as the nucleons.
- QCD is asymptotically free (i.e. easy at high energies), but non-perturbative in the low-energy sector where one also finds the phenomenon of confinement.

- QCD resulted from a systematization of the hadron spectrum (which could be shown to be representable in terms of the Lie group $SU(3)_F$; "the eightfold way") and a bold conjecture that the color charge (associated with the group $SU(3)_C$) should be gauged.

Quantum chromodynamics (QCD)

- Here we see already the use of some analogies:
  1. spin $\rightarrow$ isospin
  2. charge $\rightarrow$ hypercharge
  3. charge $\rightarrow$ color charge
- Charge and spin are well understood concepts, and so it makes sense to take things from there and to explore the formal analogy.
- One of the most important symmetries of QCD is chiral symmetry.

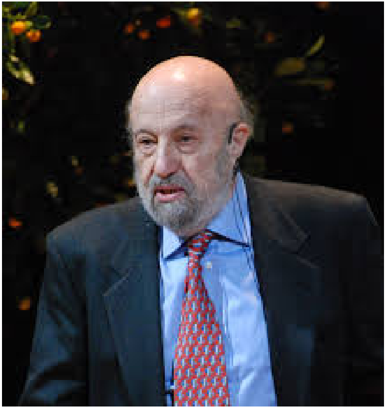
Chiral Symmetry

- In Dirac's theory, a massless fermion is described by either left or right-handed spinors: The spin is either aligned (right-handed chirality) or counter-aligned (left-handed chirality) with the momentum.
- Then the chirality is a conserved quantum number.
- A mass term in the Dirac equation breaks chiral symmetry explicitly.
- Experimentally, however, it is observed that the masses of the pseudo-scalar mesons (such as the pion) are much lighter than any of the other particles in the spectrum. The same holds for the masses of other mesons (such as the vector mesons). So chiral symmetry does not seem to hold in nature.
- This suggests that chiral symmetry is dynamically broken in QCD.
- To explore this hypothesis, one needs a good model.

III. The Nambu and Jona-Lasinio Model

Background

- Building on previous work of Yoichiro Nambu (1921–2015), Nambu and Giovanni Jona-Lasinio published two papers in 1961.
- This work led to the Nobel Prize (to Nambu) in 2008.
- Nambu studied condensed matter before switching to particle physics and was familiar e.g. with the BCS model of superconductivity.
- Nambu, as he describes in his Nobel lecture, saw an analogy between the Fermi sea and the Dirac sea and realized that something like a "gap" (to break condensed particles) could also be used to account for the masses of the nucleons without giving up chiral symmetry.
- NJL then tried to come up with a simple (toy) model that can be solved, ideally (in some approximation) analytically.

Background

- The model is non-linear, building on ideas from Heisenberg.
- The model is not renormalizable (and required a cutoff Λ). This turned out to be an advantage.
- This then led to an equation which is similar to the well-known gap equation from the BCS theory.
- Nambu explains the idea of spontaneous symmetry breaking by considering a (rotationally symmetric) elastic straight rod standing vertically on which one applies increasing pressure to squeeze it. The rod will then bend in some (possibly any) direction, and the symmetry is broken.

The Original NJL-Model

The NJL-Lagrangian

$$\mathcal{L}_{NJL} = g\left[(\overline{\Psi}\Psi)^2 + (\overline{\Psi}\mathrm{i}\gamma_5\Psi)^2\right]$$

- The NJL-Lagrangian models the interaction of the fermions as a "point" interaction.
- g is a coupling constant with dimension $1/\mathrm{energy}^2$.
- Hence, the NJL-Lagrangian is non-renormalizable.
- The NJL-Lagrangian is invariant under two transformations:
  1. $\Psi \to \mathrm{e}^{\mathrm{i}\alpha}\,\Psi$ (particle conservation)
  2. $\Psi \to \mathrm{e}^{\mathrm{i}\gamma_5\alpha}\,\Psi$ (chiral symmetry)
- These are desirable and well-motivated properties.

The Original NJL-Model

Using the mean field approximation then leads to a a self-consistent equation which has two solutions:

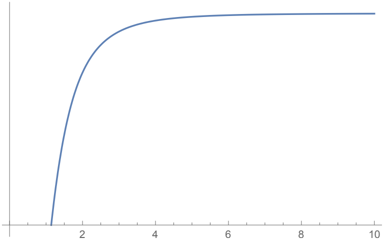
1. The solution $m = 0$ (where the symmetry holds).
2. The solution $m > 0$ follows from. . .

The NJL Gap Equation

$$\frac{2\pi^2}{g\Lambda^2} = 1 - \frac{M^2}{\Lambda^2} \log\left(1 + \frac{\Lambda^2}{M^2}\right)$$

As in the case of superconducting, an analogous condensate is formed. (The analogy helps with the interpretation.)

The Gap Equation



- The figure shows M/Λ as a function of the normalized coupling constant $g\Lambda^2/(2\pi^2)$.
- One finds that for sufficiently large values of g, there is an additional solution with $m > 0$: In this regime, chiral symmetry is dynamically broken.
- One can also add an explicit breaking of chiral symmetry by starting with a small (current) mass m_0. This leads to a smoothing of the phase transition.

Extensions of the model

- The toy model can be extended and made "more realistic", e.g. to include isospin.

The NJL-Lagrangian (with Isospin)

$$\mathcal{L}_{NJL} = g\left[(\overline{\Psi}\Psi)^2 + (\overline{\Psi}\mathrm{i}\gamma_5\tau\Psi)^2\right]$$

- With this, the model can account for parts of the (then) known particle spectrum.

From Nuclear Physics to QCD

- The original NJL model was a 'proof of concept' model. It was a toy model that could be analytically studied.
- It was not particularly successful in describing the observed hadron spectrum.
- This assessment of the model template changed with the advent of QCD.
- In QCD the lowest mass quarks are nearly massless – about 2 MeV for the up quark and 5 MeV for the down quark.
- Hence, chiral symmetry is approximately realized.
- This suggests that QCD is a much better field of application for the NJL model than nuclear physics.
- It was shown that the model could not only be extended to isospin, but also to the quark sector, taking into account three quark flavors (u, d and s).

The NJL Model and QCD

- In the late 1980ies, researchers such as Wolfram Weise repackaged the model in the context of QCD. They replaced the nucleon degrees of freedom by quark degrees of freedom.
- The model is now seen as a low-energy approximation of QCD (it is an effective field theory). This is interesting because the model does not account for confinement which is arguably the most striking a low-energy phenomenon associated with QCD.
- The model has been successful in terms of accounting for hadron data and cross sections. This might show. that the chiral dynamics is most responsible for these phenomena.
- The model is also used in the context of hot and dense nuclear matter.
- This makes sense because the deconfinement phase transition ("quark-gluon plasma") and the chiral phase transition happen at around the same critical temperature (about 175 MeV).
- The original papers of 1961 still have more than 100 citations per year.

IV. Discussion

1. The Role of Analogies

- The analogy between nuclear physics and solid state physics is certainly of heuristic value.
- But could one have predicted that it will work? Do any of the criteria of Bartha et al. apply?
- Norton's material theory does not apply as there is no *fact of analogy* connecting the source and the target.
- The analogy helps with the interpretation of the obtained qualitative result.
- It is less clear whether it also helps explaining and understanding the phenomena.

- The analogy is purely formal, and yet it does a lot of work (contra Hesse and Bartha).
- The formal analogy is not strict but liberal (Fraser 2018).
- Question: Does the analogy help to confirm the target theory?
- No!
- But it might be interesting to think about this question from the perspective of structural realism.
- This might also be interesting for our understanding of the (physical) condensates.

3. Analogue Simulation

- Analogue Simulation are special as they are not static: One intervenes in the source system to confirm a claim about the target system.
- The result of the intervention is more or less surprising, which does the epistemic work. (This is a claim contra Bartha who thinks that analogue simulations are just an example of analogical reasoning.)
- To find out whether or not this works requires us to set up a model of the epistemic situation. This model structures the discussion and allows one to see which assumptions are made that can then be supported or criticised. (Dardashti 2019)

V. Outlook

Outlook

- Analogies are everywhere in physics and analogical reasoning can be very powerful.
- Some but perhaps not all types of analogical reasoning can be reduced to other types of inductive reasoning.
- While there are many analogies uses quantum field theory. The renormalization group (Koberinski & Fraser, 2023) and spontaneous symmetry breaking are perhaps the most striking ones.
- I have discussed how ideas from superconductivity led to the NJL model, and how the NJL model could be adapted to different contexts (nuclear physics, particle physics). The model is indeed very flexible and seems to track something.

Thanks for your attention!