The Universe as an Open Quantum System

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UKRI Arts and Humanities Research Council
DFG Deutsche Forschungsgemeinschaft German Research Foundation

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Closed Model : a model is closed iff the function in the model that represents the relevant characteristic quantity is conserved under the dynamics of the model, and otherwise open.

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- In a closed system model in statistical mechanics we have a constant phase space dimension and volume measure.
- In a closed system model in quantum theory we have unitary time evolution.

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Autonomy : a model of a physical system is *autonomous* iff it includes no explicit *dynamical variables* that encode degrees of freedom other than those of the system, and there are *well-posed dynamical equations* for these variables.

- Non-autonomous since equations of motion are not well-posed due to the presence of underspecified functions leading to an underdetermined system of equations.
- Non-autonomous via the failure of the system of equations to be well-posed due to a formal breakdown of integrability

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The System-Reservoir Approach

1. Start with a classical closed system model which is such that the spatially contiguous environment (the reservoir) is given a fine-grained representation and we have autonomous, conservative dynamics.
2. Apply canonical quantization and derive a full quantum description for the system and reservoir.
3. Through a combination of limits and tracing out, derive a model for the system degrees of freedom that encodes the quantum effects of friction but does not represent the reservoir in the quantum dynamics.
4. In this manner we can derive an open system model (conservation fails) which is autonomous.

Caldeira-Leggett Model

- The derivation of the Caldeira-Leggett model deploys the system-reservoir approach with the environment explicitly modelled as a set of k non-interacting harmonic oscillators which are linearly coupled to the system (Joos, 2013, pp. 81-8).
- Autonomous, open system master equation of the form :

$$i\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}_{\mathcal{S}}, \hat{\rho}] + \frac{\eta}{2m}[\hat{q}, \{\hat{p}, \hat{\rho}\}] - i\eta \mathrm{k_B} T[\hat{q}, [\hat{q}, \hat{\rho}]] \tag{1}$$

  where $\hat{\rho}$ is the density operator for the system and $\{,\}$ is the anti-commutator and where the first term describes the standard unitary dynamics, the second term describes friction, the third one describes decoherence.
- Equation is trace-preserving (probability conserved) but does not generate a completely-positive map (Stinespring dilation theorem does not apply).

It is instructive to consider the momentum and energy generalised Ehrenfest type relations that can be derived for the CL model :

$$\frac{d}{dt}\langle\hat{q}\rangle = \frac{1}{m}\langle\hat{p}\rangle \tag{2}$$

$$\frac{d}{dt}\langle\hat{p}\rangle = -\Big\langle\frac{d}{dq}V(\hat{q})\Big\rangle - \frac{\eta}{m}\langle\hat{p}\rangle \tag{3}$$

$$\frac{d}{dt}\big\langle\hat{H}\big\rangle = \frac{2\eta}{m}\Big[\frac{k_B T}{2} - \Big\langle\frac{p^2}{2m}\Big\rangle\Big] \tag{4}$$

which equates to a frictional ‘force’ term and mean energy *dissipation towards thermal equilibrium.*

Lesson 1 : Open systems are in general systems that fail to conserve some relevant quantity. Open systems can be modelled via autonomous systems of equations that need not make explicit reference to further environmental systems.

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Dissipation and the Open Systems

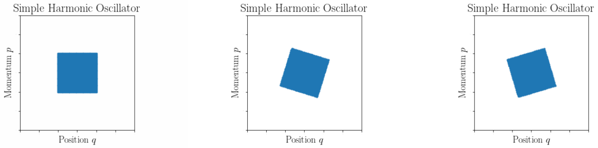
- Intuitively the idea of dissipation in the universe as a whole makes very little sense, *loss of energy to where*?
- However, in many cosmological models the energy of the universe is taken to be zero, and, moreover, at the statistical level dissipation can be represented in terms of *measure compression* rather than energy loss.
- What is more, we can motivate *empirically adequate* cosmological models which display dissipative dynamical behaviour as a *re-description of Hubble friction*.
- To understand these models we need to make a short digression into *contact* Hamiltonian systems following Bravetti et al. (2017)

Symplectic Geometry and Closed Classical Systems

- The geometric description of *closed* classical system is an (even-dimensional) symplectic geometry, (H, ω, Γ), with conservation of Liouville measure induced by the *conserved volume form* ω^n on the state space.
- The dynamics of a symplectic Hamiltonian system are given by Hamilton's equations :

$$\dot{q}^i = \frac{\partial H}{\partial p},\ \dot{p}^i = -\frac{\partial H}{\partial q} \tag{5}$$

- The system of equations describes a closed classical system where state space volume is the characteristic conserved quantity.

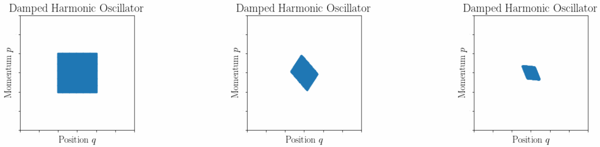


Contact Geometry and Open Classical Systems

- The geometric description of an *open* classical system is an (odd-dimensional) contact geometry $(H_c, \eta, \mathcal{C})$ with *compression* of Liouville measure induced by the *non-conserved volume form* $\nu = \eta \wedge (d\eta)^n$ on the state space.
- The dynamics of a contact Hamiltonian system are given by the contact Hamilton's equations :

$$\dot{q}^i = \frac{\partial H_c}{\partial p}, \; \dot{p}^i = -\frac{\partial H_c}{\partial q} - p_i \frac{\partial H_c}{\partial z}, \; \dot{z} = p_i \frac{\partial H_c}{\partial p_i} - H_c \tag{6}$$

- The system of equations describes an open classical system where state space volume is the characteristic non-conserved quantity.



- Example : Consider a product manifold $\mathcal{C} = \mathbb{R} \times T^{\star}\mathbb{R}^2$ with contact structure $\eta = dz - p_1 dq^1 - p_2 dq^2$. The contact Hamiltonian $H : \mathcal{C} \to \mathbb{R}$ is :

$$H(z, \boldsymbol{p}, \boldsymbol{q}) = \frac{1}{2}(p_1^2 + p_2^2) + V(\boldsymbol{q}) + \gamma z \tag{7}$$

  with $\gamma \in \mathbb{R} - \{0\}$

- The contact Hamilton equations are then given by :

$$\dot{q}^i = p_i, \ \dot{p}^i = -\frac{\partial V}{dq^i} - \gamma p_i, \ \dot{z} = (p_1^2 + p_2^2) - H, \ \text{for } i = 1, 2. \tag{8}$$

- Combining the first two equations gives the characteristic second order equation for a dissipative system :

$$\ddot{q}^i + \gamma \dot{q}^i + \frac{\partial V}{\partial q^i} = 0 \tag{9}$$

- Scaling symmetries are conformal transformations that do not change the image of dynamical curves in the state space. They can be represented as vector fields $\boldsymbol{D} \in \mathcal{X}(\Gamma)$.
- Recent work by Bravetti et al. (2023) has provided a general algorithm to apply a procedure to *quotient-out* scaling symmetries in terms of a *contact reduction* of a symplectic Hamiltonian system (H, ω, Γ) to a contact Hamiltonian system $(H_c, \eta, \mathcal{C})$.
- In simple cases this takes the form of a direct state space reduction whereby :

$$\mathcal{C} = \Gamma/\boldsymbol{D} \tag{10}$$

- Consider the flat FLRW symplectic system with variables (a, p_a, ϕ, p_ϕ). Ignore all pre-factors and set constants to one to get the Hamiltonian :

$$\mathcal{H} = a^3\left(-3h^2 + \left[\frac{p_\phi^2}{a^6} + V(\phi)\right]\right) = 0 \tag{11}$$

where $h = \frac{\dot{a}}{a}$ is the Hubble parameter.

- Define the new variable :

$$\Pi = \frac{p_\phi}{a^3} \tag{12}$$

- Re-write the Hamiltonian as :

$$\mathcal{H}_c = \mathcal{H}/a^3 = -h^2 + \Pi^2 + V(\phi) = 0 \tag{13}$$

which implies :

$$h^2 = \Pi^2 + V(\phi) \tag{14}$$

- This is the contact Hamiltonian for the contact system $(\phi, \Pi, -h)$. The first two contact Hamilton's equations give us :

$$\dot{\phi} = \frac{\partial \mathcal{H}_c}{\partial \Pi} = \Pi \tag{15}$$

$$\dot{\Pi} = -\frac{\partial \mathcal{H}_c}{\partial \phi} - \Pi \frac{\partial \mathcal{H}_c}{\partial h} = -\frac{\partial V(\phi)}{\partial \phi} - \Pi h \tag{16}$$

- This is equivalent to a second order equation in ϕ :

$$0 = \ddot{\phi} + h\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} \tag{17}$$

which we can think of as autonomous equation for a matter system with potential $V(\phi)$ and damping factor $h = \sqrt{\dot{\phi}^2 + V(\phi)}$.

- Sloan (2023) has shown that contact reduction can be applied to the class of homogenous but anisotropic Bianchi models and argued that *cosmology with scale can be seen to arise as the symplectification of a more parsimonious contact system.*
- Contact cosmologies are such that we have non-conservation of the Liouville measure and they are thus open systems in the relevant sense. What is more, the equations of motion are autonomous

Lesson 2 : There is an empirically adequate framework to model the universe as an autonomous, open classical system, based upon a scale-invariant contact dynamics, with measure compression representing cosmological dissipation.

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Three Forms of Cosmic Non-Unitarity

- **Extrinsic Canonical** : non-unitarity dynamics results from a de-parameterization of the Wheeler-DeWitt equation simply with respect to a (bad) internal clock choice (Gielen and Menéndez-Pidal, 2022)
- **Intrinsic Canonical** : non-unitarity dynamics results from a non-self-adjoint Hamiltonian with the wave packet asymptotically collapsing with probability loss and the quantum evolution is described by a contraction semigroup (Gotay and Demaret, 1983)
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Proposal

Since the CL-equation has a damped classical system as its semi-classical limit a cosmological version of the CL-equation with the modified damping factor will have the contact Friedmann equations as its semi-classical limit.

Cosmological CL-equation

$$i\hbar\frac{d\rho}{dt} \quad = \quad [\hat{\Pi}^2 + V(\hat{\phi}), \rho] - i\frac{\gamma}{\hbar}[\hat{\phi}, \{-\hat{\Pi}h, \hat{\rho}\}] \tag{18}$$

where $h = \sqrt{\hat{\Pi}^2 + V(\hat{\phi})}$ and we now take $\hat{\rho}, \hat{\Pi}, \hat{\phi}$ to be operators and adopt the ordering convention that momentum operators act to the right.

The Ehrenstest type relations can then be shown to take the form :

$$\frac{d}{dt}\left\langle \hat{\phi} \right\rangle \quad = \quad \left\langle \hat{\Pi} \right\rangle \tag{19}$$

$$\frac{d}{dt}\left\langle \hat{\Pi} \right\rangle \quad = \quad -\langle \frac{dV(\hat{\phi})}{d\phi} \rangle - \left\langle \hat{\Pi}h \right\rangle \tag{20}$$

For $V = 0$ the classical solution is straightforward. We have that $h = \Pi = \dot{\phi}$ and $\dot{\Pi} = -\Pi^2$ so the problem reduces to solving the equation :

$$\ddot{\phi} = -\dot{\phi}^2 \tag{21}$$

which is solved by $\phi = \ln(At + B)$.

Introduce the following representation of the operator algebra :

$$\hat{\phi} \equiv \mathbf{x} = x - \frac{\hbar}{2}\theta \, , \; \hat{\Pi} \equiv \mathbf{p} = p + \frac{\hbar}{2}\lambda \tag{22}$$

$$\mathbf{x}' = x + \frac{\hbar}{2}\theta \, , \; \mathbf{p}' = p - \frac{\hbar}{2}\lambda \tag{23}$$

Where we have that $[\mathbf{x}, \mathbf{p}] = i\hbar$ and $[\mathbf{x}', \mathbf{p}'] = -i\hbar$, $[x, \lambda] = [p, \theta] = i$ and all other commutators vanish.

We then have the following crucial relation :

$$\rho A(\mathbf{x}, \mathbf{p}) = A(\mathbf{x}', \mathbf{p}')\rho \tag{24}$$

CLC master equation in solvable form

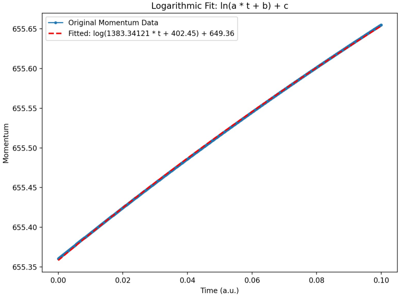
Using the quantum Hilbert Phase Space Formalism we can now re-write our master equations as a Schrödinger-type Equation of the form :

$$\begin{aligned}
i\hbar\frac{d\rho}{dt} &= \frac{1}{2m}[\mathbf{p}^2,\rho] - i\frac{\gamma}{\hbar}[\mathbf{x},\{-\mathbf{p}^2,\rho\}] \\
&= \frac{1}{2m}\Big(\mathbf{p}^2\rho - \rho\mathbf{p}^2\Big) - i\frac{\gamma}{\hbar}\Big(-\mathbf{x}\mathbf{p}^2\rho - \mathbf{x}\rho\mathbf{p}^2 + \mathbf{p}^2\rho\mathbf{x} + \rho\mathbf{p}^2\mathbf{x}\Big) \\
&= \frac{1}{2m}\Big(\mathbf{p}^2 - \mathbf{p}'^2\Big)\mid\rho\rangle - i\frac{\gamma}{\hbar}\Big(-\mathbf{x}\mathbf{p}^2 - \mathbf{x}\mathbf{p}'^2 + \mathbf{p}^2\mathbf{x}' + \mathbf{p}'^2\mathbf{x}'\Big)\mid\rho\rangle \\
&= \Big(\frac{\hbar}{2m}p\lambda - i\gamma p^2\theta - 2\gamma p + \gamma\hbar\lambda + i\frac{\hbar^2}{2}\lambda^2\theta\Big)\mid\rho\rangle
\end{aligned}$$

Which we can solve using the numerical approach of Cabrera et al. (2015) wherein we evolve the Wigner function at each time step by multiplication in the appropriate Fourier space.

Results

We can plot $\left\langle \hat{\phi}(t) \right\rangle$ from the numerical solution against these classical solution.



Prospectus

The model thus reproduces the classical solutions for the free case – which is what we should expect from the analytic treatment in any case.

The problem in extending this approach to the case with a potential i.e. inflation, since we have square root operators. Not clear how to move forward.

Our current plan is to explore the quantum phenomenology of the free case, and then numerically study the inflationary model via the semi-classical properties of the moment expansions.

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