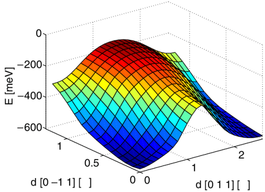
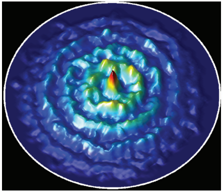
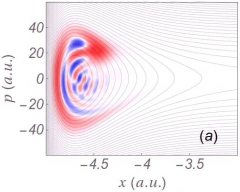
Three Roads From Quantum Analogy







Karim Thébault (Bristol)
(based upon work with Pete Evans, Nick Huggett, James Ladyman, David Sloan, and Will Wolf)

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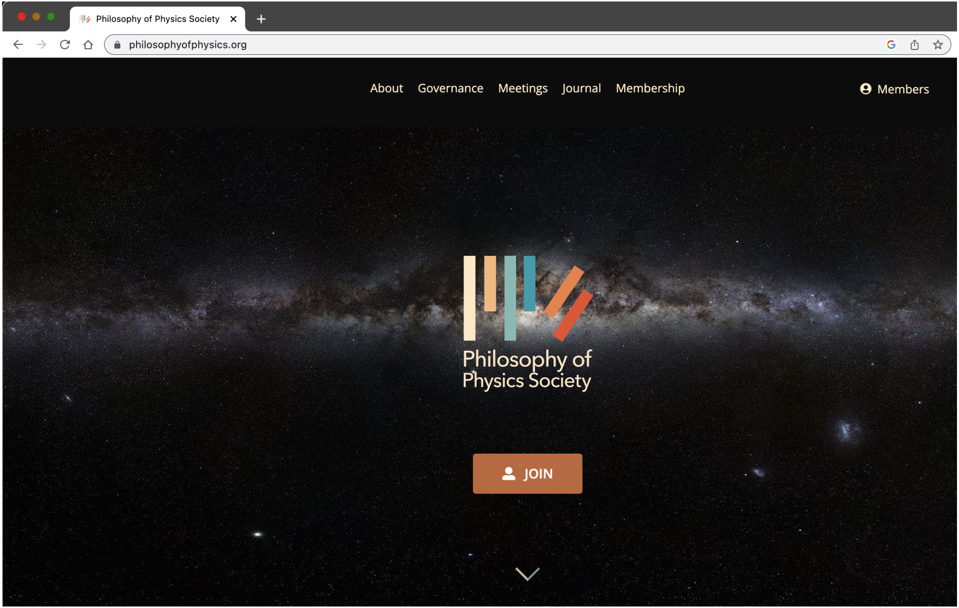
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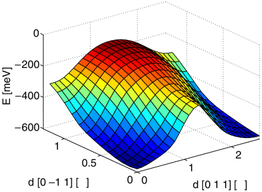

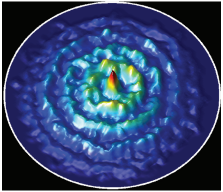
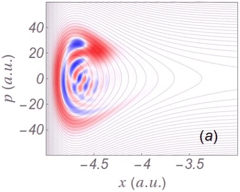

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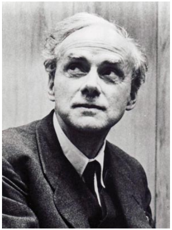
Overview

- Analogy plays a wide variety of roles in the context of physical theory. Here I want to focus on three examples of the *use* of quantum analogies in different *methodological* contexts.
- I will consider quantum analogies in three particular contexts:
  1. Born-Oppenheimer **formal heuristic analogies** in simple models for a Timeless Wheeler-DeWitt Cosmology;
  2. Bouncing oil-droplets **illustrative analogies** in Pilot-wave approaches to Quantum Theory;
  3. Caldeira-Leggett **physical heuristic analogies** in efforts to derive models for a Dissipative Open Quantum Cosmology.
- In each case I will consider the potential for insight and confusion within the interpretation of the analogies and try and draw some general lessons.

Roadmap

1. *From Quantum Chemistry to Quantum Cosmology*
2. *From Bouncing Oil-Droplets to Pilot-wave Quantum Theory*
3. *From Quantum Dissipation to Open Quantum Cosmology*

1. *From Quantum Chemistry to Quantum Cosmology*

Wheeler-DeWitt Cosmology

- The gravitational Wheeler-DeWitt equation is a 'semi-mathematical' expression for wavefunction of the universe in quantum gravity.
- The equations results from informal application of the Dirac constraint quantization algorithm to the Hamiltonian formulation of general relativity.
- Famously the equation does not contain any extrinsic temporal structure.

Wavepackets in Minisuperspace

Consider the Wheeler-DeWitt quantization of an finite dimensional symmetry reduced mini-superspace FLRW-type universe with spatial curvature k, scale factor a, homogeneous scalar field ϕ with mass m, and no cosmological constant (Kiefer 1988, 2012):

$$\left[\frac{2}{3\pi m_p^2}\frac{\partial^2}{\partial^2\alpha} - \frac{\partial^2}{\partial\phi^2} - \frac{3\pi m_p^2}{2}ke^{4\alpha} + m^2e^{6\alpha}\phi^2\right]\psi(\alpha,\phi) = 0 \qquad (1)$$

where $\alpha = \ln a$, and m_p is the Planck mass. This corresponds to a 'frozen' time independent equation for a **single zero energy eigenstate**.

Emergent Temporal Structure

- In order to derive an effective internal temporal structure we need to be able to separate a degree of freedom that plays the role of a clock from the other degrees of freedom.
- This requires us to be able to approximately neglect the coupling between the 'clock' degree of freedom and the dynamical degrees of freedom.
- Kiefer shows that this is possible for the system described by Equation (1) and the choice of α as the clock variable because the *very large* value of m_p.

Born-Oppenheimer Cosmology

- For each α we define a reduced Hamiltonian of the form:

$$H_\alpha = -\frac{\partial^2}{\partial \phi^2} - ke^{4\alpha} + m^2 e^{6\alpha} \phi^2 \tag{2}$$

which is such that

$$H_\alpha \phi_n(\alpha;\phi) = E_n(\alpha)\phi_n(\alpha;\phi). \tag{3}$$

- It is assumed that each of the n-parameterised families $\phi_n(\alpha;\phi)$ are "eigenfunctions" each with "eigenvalues" $E_n(\alpha)$, all of which vary only "adiabatically" with the parameter α.[1]

---

[1] N.b. the scare quotes are due to the fact that a little more work is needed to make these objects rigorously well-defined, see Jecko 2014).

Born-Oppenheimer Cosmology

- We then make the separation ansatz:

$$\psi(\alpha,\phi) = \sum_n C_n(\alpha)\phi_n(\alpha,\phi) \tag{4}$$

  and insert pack into our full Wheeler-DeWitt Equation (1) and consider an “orthonormal” scalar product with states $\phi_l^\star$.

- We then neglect terms of the form $\frac{\partial \phi_n}{\partial \alpha}$ precisely because the variation of $\phi_n(\alpha;\phi)$ with respect to α is assumed to be zero.
- One can simply check by explicit calculation whether adiabatically holds once the trial solutions have been found. Kiefer identifies the regime of validity in terms of the excitation level n of the reduced eigenvalue problem (3) .

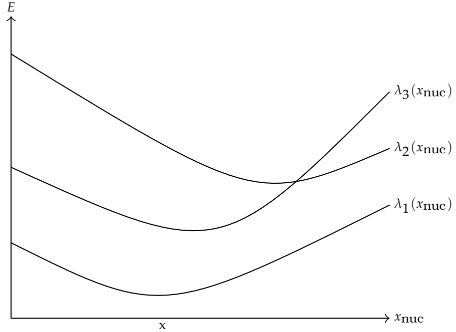
Born-Oppenheimer Approximation

- There is a **partial formal analogy** between the method used for solving the Equation (1) and the Born-Oppenheimer method used in molecular quantum chemistry via a reduced equation of the form:

$$\left(T_e + W(x_{\text{nuc}})\right)\psi_a(x_{\text{nuc}}; x_{\text{elc}}) = \lambda_a(x_{\text{nuc}})\psi_a(x_{\text{nuc}}; x_{\text{elc}}), \qquad (5)$$

- In that context, the crucial assumption is that in a stable molecule the nuclei are approximately localized, in a quantum state in which their kinetic energy is much smaller than the electron kinetic energy (though not zero).
- The solutions we are looking for correspond to the energy levels of the light subsystem being widely separated with respect to the kinetic energy of the heavy subsystem and this correspond to considering **distinct energy eigenstates** for the total system (very much unlike in WdW cosmology)

Born-Oppenheimer Approximation



- Quantitively this corresponds the *gaps* between the "eigenvalues" of the "electronic" part of the wavefunction $\psi_a(x_{\text{nuc}}, x_{\text{elc}})$ being much greater than the *values* of nuclear kinetic energy T_{nuc}.
- From that assumption one can demonstrate that the approximate validity of the molecular form of the Born-Oppenheimer separation ansatz and the adiabatic approximation.

No Time for Time from No-Time

Eugene Y. S. Chua and Craig Callender*†

Programs in quantum gravity often claim that time emerges from fundamentally timeless physics. In the semiclassical time program, time arises only after approximations are taken. Here we ask what justifies taking these approximations and show that time seems to sneak in when answering this question. This raises the worry that the approach is either unjustified or circular in deriving time from no-time.

Dodgy Analogy?

[Born-Oppenheimer] applies in cases in which heavier subsystems are known to change slowly in time with respect to lighter subsystems. That is why mass matters. Heavier subsystems have significantly different characteristic dynamical timescales - timescales over which "the parameters of the system change appreciably" - and can be said to be adiabatic, with respect to the lighter subsystems.

Dodgy Analogy?

Because the BO approximation is so widely and successfully used, and because it initially seems to be about mass (not time!), it may be imported into derivations without considering whether the conditions warrant its use in a new application. [...] Either the mass scales relevant here are proxies for time scales or not. If they are then we face circularity; if they are not, then we have no clear means of assessing whether BO is even applicable in this situation.

Dodgy Analogy?

Because the BO approximation is so widely and successfully used, and because it initially seems to be about mass (not time!), it may be imported into derivations without considering whether the ***conditions warrant its use in a new application.*** *[...] Either the* ***mass scales relevant here are proxies for time scales*** *or not. If they are then we face circularity;* ***if they are not, then we have no clear means of assessing whether BO is even applicable in this situation****.*

Claim. *The Born-Oppenheimer approximation uses mass scales as proxies for timescales. Applications of the approximation do not make implicit use of timescales are unjustified.*

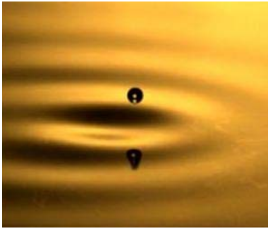
Three Problems

- First, note that the BO approximation is 'adiabatic' in the sense that ψ changes 'slowly' with respect to x_{nuc}, not time. In fact, **both** the molecular and Wheeler-DeWitt versions of Born-Oppenheimer are justified without reference to an extrinsic time.
- Second, note that the formal structure of the Kiefer's derivation is that of an *ansatz*: a trial form of solution to a differential equation is assumed and then tested for consistency. Best to think of the (non-exact) formal analogy with molecular BO as a **heuristic** for finding the form of the ansatz.
- Third, the form of criticisms about "warranted use" of a model conflates formal with physical analogy: the physical justification of an idealization within a model need not be the same when the formal structure of the model is **modified and reinterpreted within a different context** (cf. Bradley and Thébault 2018).

Lesson 1: *Scientists use formal analogies to transfers both model structure and equation solution heuristics from one context to another. Sometimes they also transfer the intuitive story used to explain justification of idealizations within the model. However, the devil is in the formal details and philosophers of science need to attend to the maths not the simply accompanying it before trying to critical engage with potential justificatory problems.*

2. *From Bouncing Oil-Droplets to Pilot-wave Quantum Theory*

Bouncing Oil-Droplets



- In 2005, a team in Paris Diderot University led by Yves Couder and Emmanuel Fort discovered that an oil droplet bouncing on a vibrating fluid surface can be made to 'walk' horizontally across the surface.
- These 'walkers' display a kind of wave-particle duality: the bouncing droplet is self-propelled by interacting with the surface waves it creates.

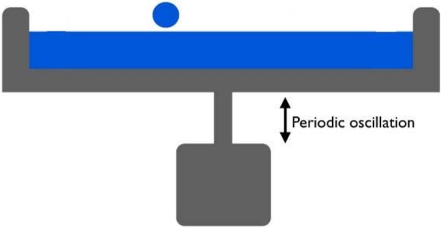
Bouncing Oil-Droplets

- Subsequent experiments from both the team in Paris and an associated team led by John Bush at MIT have since demonstrated a range of behaviour that is typically considered to be quantum.
- For example, single and double slit diffraction and interference, quantised orbits of bound state pairs, phenomena that look analogous to quantum tunnelling, Schrödinger evolution of probabilities, and Zeeman splitting.
- Entanglement is notably absent from this list.

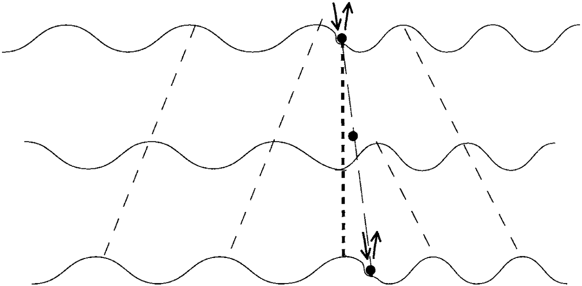
Bouncing Oil-Droplets

- Consider a small, shallow rectangular bath oriented horizontally, filled with a layer of silicon oil, and parametrically driven from below by a low frequency generator to vibrate vertically.
- By piercing the fluid surface with a pin and then withdrawing quickly, a small oil droplet can be created which, due to the forced vibrations, bounces upon the fluid surface.

The Paris experiments

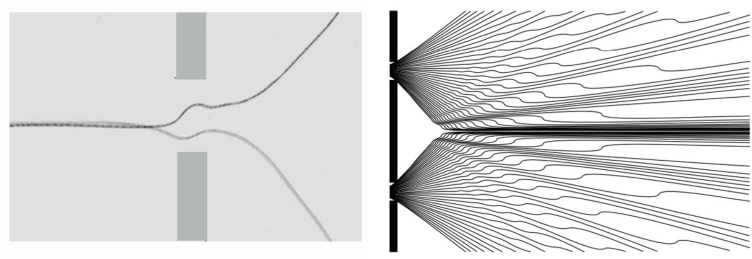


Walkers



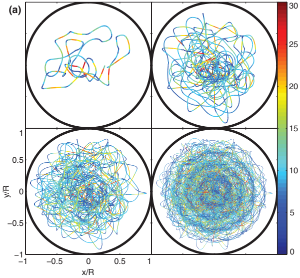
Each time the drop hits the surface a new dip forms, shifted from the trough that would have been formed by the evolution of the previous wave-packet. The resulting wave is thus the superposition of waves generated by a source that is slightly displaced at each jump. (Protière et al. 2006, p.92)

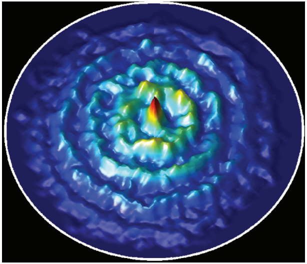
From path memory to pilot-wave dynamics



This interplay between the droplet motion and its associated wave field makes it a macroscopic implementation of a pilot-wave dynamics. (Couder and Fort 2012, p.2)

Probability wave





We can thus understand the probability distribution as being a manifestation of the characteristics of the underlying trajectories. In the confined circular geometry, the pilot-wave dynamics tends to drive the walker along circular orbits with radii corresponding to maxima of the cavity mode amplitude (Harris et al. (2013): p.011001-4)

What is going on?

- To explain this correspondence it has been variously suggested that the fluid mechanical system provides a single-particle classical model of de Broglie's idiosyncratic 'double solution' pilot wave theory of quantum mechanics
- Borghesi (2017) has constructed a classical fluid dynamical model for the walker system and shown that there is a partial isomorphism between structural equations describing the concretion and elastic medium the model and the u-waves in de Broglie's pilot wave theory.
- Note, however, that this this partial isomorphism establishes a correspondence between empirical terms in Borghesi's model and key *extra-empirical* terms in de Broglie's pilot wave theory, including the quantum phase and the pilot wave itself.

Illustration vs Simulation

- Analogue illustration, unlike analogue simulation, is not a form of ‘material surrogacy’, in which source empirical phenomena in a system of one kind can be understood as ‘standing in for’ target phenomena in a system of another kind.
- Rather, analogue illustration leverages a correspondence between certain empirical phenomena displayed by a source system and aspects of the ontology of a target system.

Illustration vs Simulation

- On the one hand, this limits the potential inferential power of analogue illustrations, but, on the other, it widens their potential inferential scope.
- In particular, through analogue illustration we can learn, in the sense of gaining how-possibly understanding, about the putative ontology of a target system via an experiment (cf. Reutlinger et al., 2017).

Lesson 2: *The walker experiments do not give empirical support of any kind for the pilot-wave interpretation of quantum theory. Rather they are best understood as analogue illustrations of certain aspects of the ontology of the theory. As such, their value is principally in terms of mediating how-possibly understanding of phenomena such as wave-particle duality. In this regard analogue illustrations function much like a material counterpart of toy models.*

3. *From Quantum Dissipation to Open Quantum Cosmology*

Analogy is one of the cognitive strategies available for creative discovery from which scientific models result (Bailer-Jones (2009, p.56)

Physical Heuristic Analogies

- In our examples so far we have considered the use of a formal analogy to transfers equation solution heuristics from one context to another and the use of an analogue illustration to gain understanding of the ontology of a theory.
- The final example of an active use of analogy in science I want to consider is in the context of a physical analogy being used as a heuristic for finding a new approach to modelling cosmology as an open system.

The Universe Cannot be Open!?

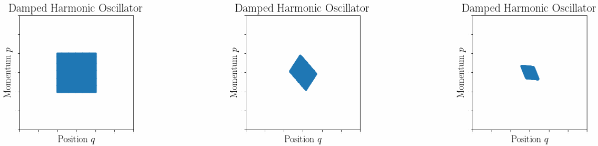
The idea of the universe as *analogous* to an open quantum system seems absurd: open systems are standardly understood to be coupled to an environment to which they dissipate entanglement or energy. The universe has no environment?!

Classical Dissipation

- The second contact Hamilton equation for a damped oscillator is:

$$\dot{p}^i = -\frac{\partial V}{dq^i} - \gamma p_i \tag{6}$$

- Contact dynamics allows us to give a general definition of *autonomous* classical open systems in terms of measure compression.



Quantum Dissipation

Caldeira-Leggett model:

$$i\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}_{\mathcal{S}}, \hat{\rho}] + \frac{\eta}{2m}[\hat{q}, \{\hat{p}, \hat{\rho}\}] - i\eta \mathrm{k_B} T[\hat{q}, [\hat{q}, \hat{\rho}]] \tag{7}$$

The first term describes the standard unitary dynamics, the second term describes 'quantum friction', the third one describes decoherence.

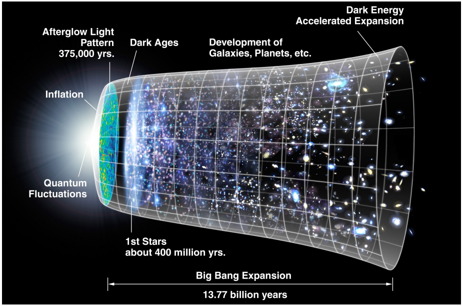
Caldeira-Leggett Model

It is instructive to consider the generalised Ehrenfest type relation for momentum that can be derived for the CL model:

$$\frac{d}{dt}\langle\hat{p}\rangle = -\Big\langle\frac{d}{dq}V(\hat{q})\Big\rangle - \frac{\eta}{m}\langle\hat{p}\rangle \tag{8}$$

This equates to a frictional 'force' term that matches the second contact Hamilton equation for a damped oscillator.

Hubble Friction



The Friedmann equations in scale-invariant variables re-describe the expansion of space as the evolution of matter with a frictional force with the formal structure of a contact system with measure compression (energy constrained to zero).

Open Quantum Cosmology

- Cosmological Caldeira-Leggett:

$$i\hbar\frac{d\rho}{dt} = [\hat{\Pi}^2 + V(\hat{\phi}), \rho] - i\frac{\gamma}{\hbar}[\hat{\Pi}, \{-\hat{\Pi}h, \hat{\rho}\}] \tag{9}$$

where $h = \sqrt{\hat{\Pi}^2 + V(\hat{\phi})}$.

- The generalised Ehrenfest type relations for this equation match the scale-invariant version of the Friedmann equations.

Lesson 3: *The heuristics of a physical analogy can be surprising when the role of key concepts, such as dissipation, changes between contexts. The idea of open systems quantum cosmology need not be nonsensical since although there may be no environment in quantum cosmology dissipation can be made sense of in different terms.*

- Finding Time for Wheeler-DeWitt Cosmology, with Nick Huggett, 2023 https://philsci-archive.pitt.edu/22669/
- What can bouncing oil droplets tell us about quantum mechanics?, with P. Evans, European Journal for Philosophy of Science 10(49) 2020. https://philsci-archive.pitt.edu/17336/
- Open Systems and Autonomy, with James Ladyman 2024 https://philsci-archive.pitt.edu/23701/